Light hadron spectrum—MILC results with the Kogut-Susskind and Wilson actions*

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We present the current status of our ongoing calculations of the light hadron spectrum with both Kogut-Susskind (KS) and Wilson quarks in the valence or quenched approximation. We discuss KS quarks first and find that the chiral extrapolation is potentially the biggest source of systematic error. For the Wilson case, we focus on finite volume and source size effects at $6/g^2 = 5.7$. We find no evidence to support the claim that there is a finite volume effect between $N_s = 16$ and 24 of approximately 5%.

1. INTRODUCTION

It has been some 15 years since the first calculations of the light quark spectrum were done on computers capable of sustaining about 1 Megaflop [1]. Today, computers exist that can achieve over one million times that speed, and if we can just get enough time on them, we should be able to control carefully the systematic errors arising from finite volume, a quark mass that is heavier than in nature, and a nonzero lattice spacing. We are certainly close to that goal within the valence or quenched approximation (even without going to improved lattice actions), if not yet with dynamical quarks. Here we will concentrate on some recent results of the MILC Collaboration within the quenched approximation. We are also doing calculations with dynamical KS quarks that are covered in another talk at this conference [2]. One of us has recently reviewed the light quark spectrum at Lattice 96 [3] and an overview of other groups' work can be found in the proceedings. Additional details are available at the URL `http://physics.indiana.edu/~sg/lat96_spectrum.html`.

In the rest of this paper, we list the lattices that were used in our KS calculations and then discuss the sources of systematic errors. We first discuss what volume is needed to avoid finite size effects, then the quark mass (or chiral) extrapolation required and, finally, the lattice spacing extrapolation. We have found that the chiral extrapolation is the most delicate issue. We then turn to recent calculations with Wilson quarks.

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Table 1
Lattices used for quenched Kogut-Susskind calculations

$6/g^2$	size	#
5.54	$16^3 \times 32$	200
5.7	$8^3 \times 48$	600
5.7	$12^3 \times 48$	400
5.7	$16^3 \times 48$	400
5.7	$20^3 \times 48$	200
5.7	$24^3 \times 48$	200
5.85	$12^3 \times 48$	200
5.85	$20^3 \times 48$	200
5.85	$24^3 \times 48$	200
6.15	$32^3 \times 64$	135

We discuss the same sources of systematic errors and how our calculations can help shed light on the crucial issues. At the time of the conference, our running was not completed. We present some preliminary results and outline what we will be able to study when all of our hadron propagators are analyzed.

2. THE QUENCHED KS SPECTRUM

2.1. Ensemble and calculational details

Table 1 lists the couplings, volumes and number of lattices used in our Kogut-Susskind calculation. To generate the gauge configurations, we use a combination of the overrelaxed algorithm [4] and the pseudo heat bath algorithm [5] in the ratio of 4 sweeps to 1. Each lattice is separated from the previous one by 200 such updates, or a total of 800 overrelaxed and 200 heat bath sweeps. We have looked at five quark masses for each gauge coupling. The heaviest quark mass is 16 times the lightest. For $6/g^2 = 5.54$, the range of quark mass am_q is 0.02–0.32. At 5.7, it is 0.01–0.16. The same range is used at 5.85, and the range is reduced to 0.005–0.08 at 6.15.

To calculate the hadron propagators, we use wall sources. On the source time slice, all sites with all spatial coordinates even are set to one, all other sites to zero. On each lattice, we have a source every 8 time slices, *i.e.*, we have four sets of propagators per lattice for $6/g^2 = 5.54$, six for 5.7 and 5.85 and 8 for 6.15. We have over 1000 propagators for every case except for 5.54. Propagators on each lattice with different source times are blocked together for further analysis.

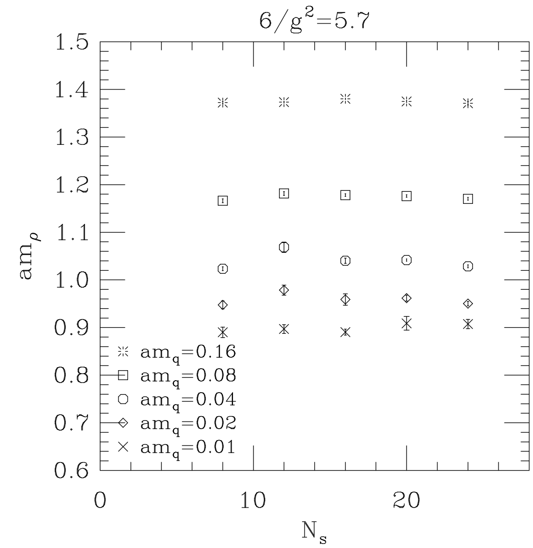

Figure 1. The rho mass as a function of lattice size N_s for various quark masses with $6/g^2 = 5.7$.

2.2. Finite size effects

It is well known that if the lattice size is too small hadrons get squeezed and their masses increase [6]. For coupling 5.7 we have five lattice sizes 8, 12, 16, 20 and 24. In Figs. 1 and 2, we display our result for the nucleon and rho masses as a function of spatial size in lattice units. Over this range of sizes we do not see any noticeable trend for the rho masses. For the smaller quark masses, the nucleon mass appears to be decreasing with increasing size at $N_s = 8$ and perhaps also at 12. If we use the rho mass (extrapolated to zero quark mass) to set the scale, the lattice spacing a is 0.23 fm and this range of lattice sizes corresponds to 1.84–5.52 fm.

At 5.85, we have studied three spatial sizes $N_s = 12$, 20 and 24. (See Figs. 3 and 4.) Again,

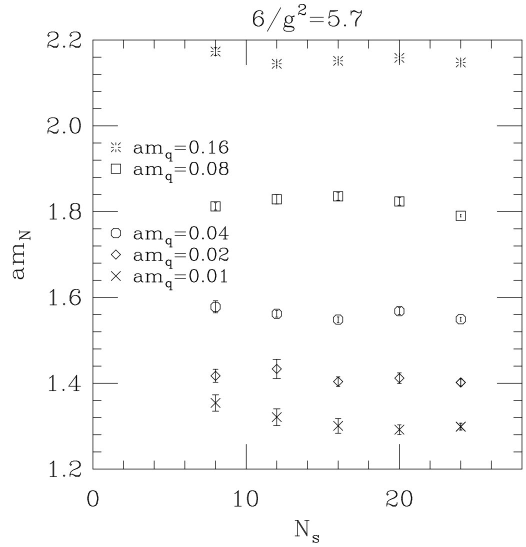

Figure 2. The nucleon mass as a function of lattice size N_s for various quark masses with $6/g^2 = 5.7$.

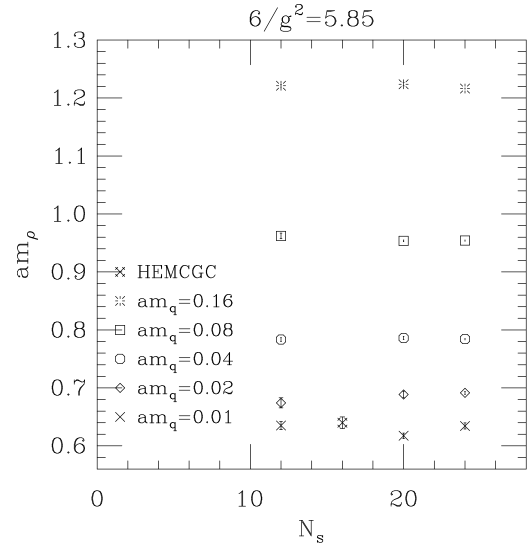

Figure 3. The rho mass as a function of lattice size N_s for various quark masses with $6/g^2 = 5.85$.

we see a similar picture with the nucleon showing a discernible effect for the lightest two quark masses. With a lattice spacing of about 0.15 fm, these sizes correspond to 1.8, 3.0 and 3.6 fm. With some evidence of where finite size effects at 5.7 and 5.85 become small, we can now appeal to the fact that this effect is physical. We argue that at 6.15 we have a large enough box size that we can safely ignore finite volume corrections. The lattice spacing at 6.15 is about 0.085 fm, so our box size is about 2.7 fm.

2.3. Chiral extrapolations

We have found that the chiral extrapolation is the most delicate issue in controlling the possible systematic errors. The extrapolation is complicated by several factors. First, the chiral expansion is a small mass expansion, but the most precise results we can generate are for large quark mass. As we reduce the quark mass, the statistical errors in the hadron masses grow. We would like to start our chiral fits in the small mass region where they are simplest and most reliable. We could then include results for heavier masses and see when higher order terms in the expansion are required. Instead, we have to apply our fits in a region where there may be large contributions from higher order terms. If our hadron masses are not very precise, or our range of quark mass is not very wide, a simple linear fit will be possible, but it may have little to do with the true low quark mass limit. The second complication is due to the fact that quenched chiral perturbation theory ($Q\chi PT$) [7] is different from ordinary chiral perturbation theory (χPT) that applies to the real world [8]. The differences between quenched and ordinary chiral perturbation theory are based on continuum rather than lattice calculations. The size of the additional terms is not well determined and they have not been clearly seen in the lattice results [9]. The third complication involves possible systematic effects in fitting the hadron propagators. The propagators must not be fit too close to the source plane or we may not be in the asymptotic region. However, there is a strong inducement in terms of statistical accuracy to fit near the source plane. With the wall sources we use, and with Gaussian sources of moderate size,

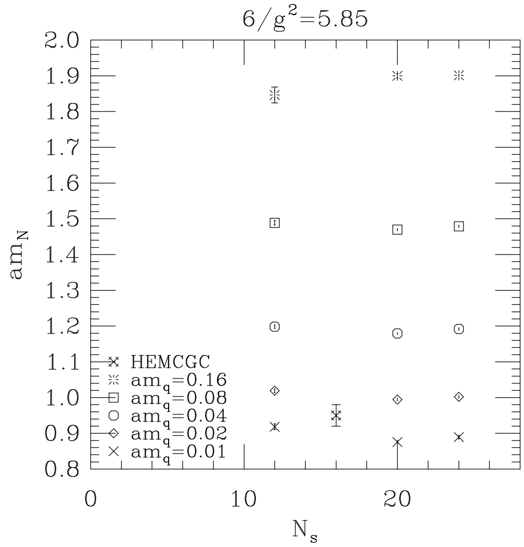

Figure 4. The nucleon mass as a function of lattice size N_s for various quark masses with $6/g^2 = 5.85$.

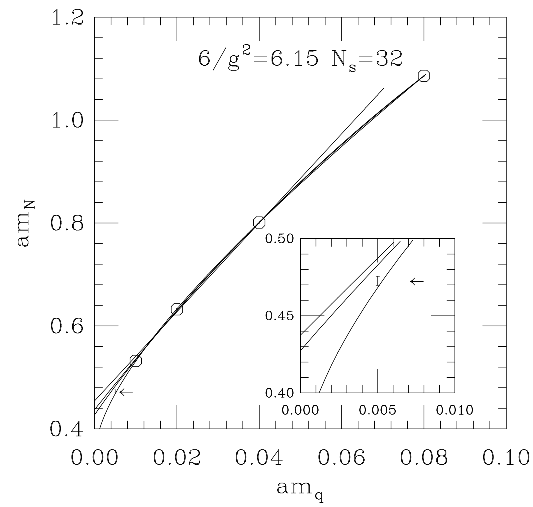

Figure 5. The chiral extrapolation for the nucleon with $6/g^2 = 6.15$. Octagons denote the points included in the fit. An arrow points to the lightest mass result which was not part of the fit.

the masses approach their asymptotic value from below. Lighter mass channels can be fit closer to the source plane because the heavier states decay away relatively more quickly; however, we must try to guard against systematic bias (from fitting too close to the source) that would make the hadron masses drop too quickly as the quark mass decreases.

We have five quark masses in each of our calculations. We cannot include too many higher order terms in the chiral expansion and still have any degrees of freedom left in the fit. In Fig. 5, we display the nucleon mass and some simple fits. There is clear curvature in the data. In fact, no three points can be adequately fit by a straight line. Non-linear terms are required in the chiral fit, and since the error on the lightest mass point is largest, we attempt some nonlinear, three parameter fits to the four heaviest masses. Each curve includes one additional power of quark mass. Looking at small quark mass, the upper two curves include $m_q^{3/2}$ or m_q^2. These are higher order terms that occur in ordinary χPT. The lowest curve includes the power $m_q^{1/2}$, a term that only appears in $Q\chi PT$. In this case, the curve with the square root very nicely touches the error bar of the point that was not part of the fit.

It is perhaps fortuitous that one of our three parameter fits touches the point we did not include in the fit. When we try to fit our other cases, we find that the three parameter fits do not work well in all cases. We have considered a dozen different fitting functions with up to four parameters. They are listed in Table 2. We have tried fitting our three lightest points, our four heaviest points and our four lightest points, in addition to fitting all five points. In Fig. 6, we show fits to the four heavy masses with squares, the four light masses with crosses and all five masses with octagons. The size of the plotting symbols is proportional to the confidence level of the fit. Fits to the three light masses are shown with no plotting symbol since most of them have no degrees of freedom. (Many other fits have such poor confidence levels that their plotting symbols are invisible.) When fitting the four lightest or four heaviest points we only considered fits with up to three parameters. We note that different fitting functions can give widely varying values. Look-

Table 2
Our fitting functions

Fit 1:	$M + am^{1/2}$
Fit 2:	$M + am^{1/2} + bm$
Fit 3:	$M + am^{1/2} + bm + cm^{3/2}$
Fit 4:	$M + am^{1/2} + bm + cm^2$
Fit 5:	$M + am$
Fit 6:	$M + am + bm^{3/2}$
Fit 7:	$M + am + bm^2$
Fit 8:	$M + am + bm^{3/2} + cm^2$
Fit 9:	$M + am + bm \log m$
Fit 10:	$M + am^{1/2} + bm + cm \log m$
Fit 11:	$M + am + bm^2 \log m$
Fit 12:	$M + am + bm^2 + cm^2 \log m$

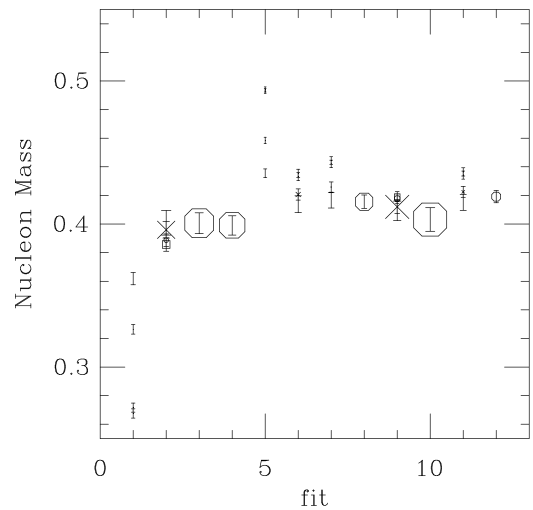

Figure 6. The nucleon mass for $6/g^2 = 6.15$ extrapolated to the physical quark mass (as determine by m_π/m_ρ) for various fits. The symbol sizes are proportional to the confidence level of the fits. The meaning of the symbols is described in the text.

ing at fit 5, the linear one, we see that the errors of the fits are small, but the result varies greatly depending upon which data is included in the fit. The confidence levels of these fits are unacceptable. If one neglects that fact, the systematic error can greatly exceed the quoted statistical error. The fits with square roots all have low values for the extrapolated mass and have large errors because of the large derivative of the square root near zero quark mass.

We see that we have a number of octagons and two crosses of reasonable size. Thus, we must decide which higher order terms to use in our fits. In Table 3 we give the combined confidence levels of our fits. Since this is a quenched simulation, all the hadron masses on the lattices for any given coupling and volume are correlated. We estimate the correlation matrix from a jackknife analysis of the hadron propagators. The simulations with different volumes or couplings are independent, so it is easy to compute the combined confidence level of the chiral fit by summing the χ^2 and degrees of freedom from each case and then computing the confidence level from the sums. For each fitting form we report two combined confidence levels. The column labeled “Biggest volume” includes the biggest volumes at $6/g^2 = 5.7, 5.85$ and 6.15. The other column also includes $N_s = 20$ at 5.7 and 5.85. We did not include the 5.54 results in this analysis because it is quite a strong coupling, and we want our chiral fits to reflect the proper continuum form.

What we find from our table of confidence levels, is that for the nucleon, five fits have reasonable confidence levels. These are the forms that contain four free parameters. The best fit, number 8, includes both $m^{3/2}$ and m^2, two terms that come into ordinary chiral perturbation theory. However, with a confidence level of 0.18, it is not markedly better than fits 3 or 4 that have confidence level 0.12 and 0.13, respectively. Both these fits have an $m^{1/2}$ term that is characteristic of $Q\chi PT$. Fits 10 and 12 round out the set of five fits. Fit 10 has two terms that only appear in quenched chiral perturbation theory. Fit 12 has terms from ordinary chiral perturbation theory.

The fits for the rho are more problematic. None of the fits does an acceptable job fitting all the results. In neither column is any combined confidence level above 10%. As for the nucleon, the four parameter fits, numbers 3, 4, 8, 10 and 12 seem to do the best job. (The logarithm in fit 10 is not supposed to occur even in $Q\chi PT$.) We

Table 3
Combined confidence levels of fit

Fit	All 5 cases	Biggest volume
	Nucleon Jackknife fits	
1	5.81e-273	1.75e-265
2	4.66e-08	3.42e-07
3	1.24e-01	8.69e-02
4	1.30e-01	1.05e-01
5	0.00e+00	0.00e+00
6	3.02e-10	3.25e-08
7	1.06e-24	6.88e-17
8	1.83e-01	1.99e-01
9	1.05e-02	2.28e-03
10	9.95e-02	6.25e-02
11	1.52e-07	1.85e-07
12	8.38e-02	1.51e-01
	Rho Jackknife fits	
1	0.00e+00	0.00e+00
2	5.81e-33	2.60e-33
3	1.66e-02	7.46e-03
4	1.91e-02	6.92e-03
5	0.00e+00	1.26e-282
6	1.19e-05	1.65e-06
7	2.08e-04	1.66e-04
8	4.82e-02	2.57e-02
9	1.73e-12	8.15e-14
10	4.58e-03	2.57e-03
11	1.05e-07	1.10e-08
12	5.89e-02	3.54e-02

have adjusted some of our rho propagator fitting ranges to be further from the source. This should help to avoid bias from not being in the asymptotic region, and it also increases the error in the particle masses, which should help in our chiral fits. Nonetheless, we do not get good chiral fits. Whether this is a statistical fluctuation, an indication that we are underestimating our errors, a manifestation of the fact that the rho is an unstable particle, or some other problem, we have not yet determined.

It is unfortunate that we cannot determine from the confidence levels of the fits, which is best. It is particularly important to determine whether or not $m^{1/2}$ should be included since it makes a big difference in the extrapolated value. Of course, it is also an artifact of quenching.

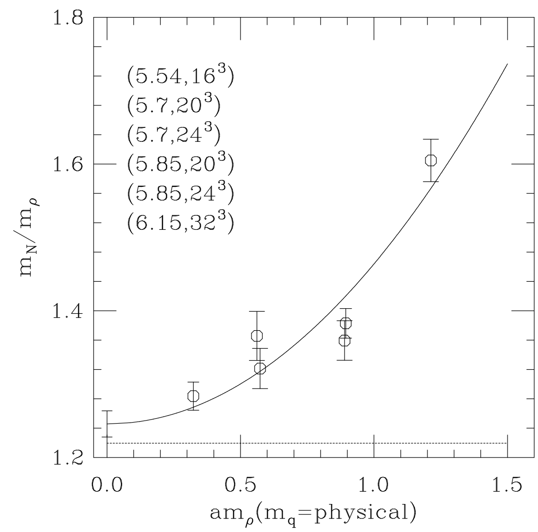

Figure 7. m_N/m_ρ at the physical quark mass *vs.* am_ρ. The horizontal line is drawn at the physical value. The error bar at $am_\rho = 0$ shows the error in the extrapolated value.

2.4. Lattice spacing dependence

The next issue to deal with is the extrapolation to zero lattice spacing. Because of the difficulty in deciding which chiral fit is best, we have considered all five of the favored possibilities for both rho and nucleon. We should mention that although the pion fits don't always have acceptable confidence levels, the errors in the pion mass are so small ($\approx< 10^{-4}$) that even a fit with poor confidence level can be very close to our values.

For Kogut-Susskind quarks with the Wilson gauge action, both parts of the action have error of order a^2. We fit the a dependence to the form

$$C + ba^2 .$$

In Fig. 7, we show one example of the extrapolation in a where we have picked fit 8 for both rho

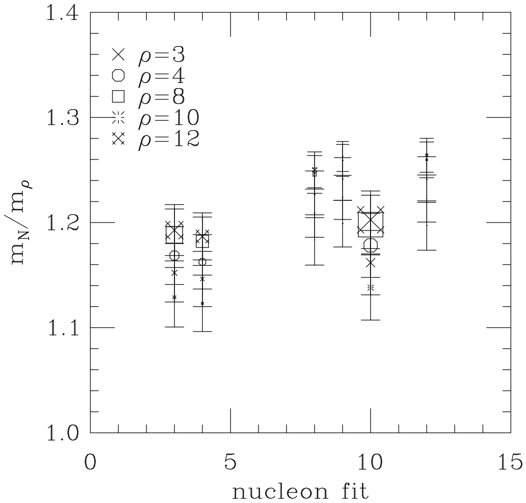

Figure 8. m_N/m_ρ extrapolated to $a = 0$ *vs.* the fit used in the chiral extrapolation. The size of the plotting symbol is proportional to the confidence level of the fit to the a dependence.

and nucleon. The octagons are for the extrapolation to the physical quark mass as determined by m_π/m_ρ. Only the largest volumes are used to fit the a dependence. The confidence level of the fit is 5.5%. In Fig. 8, we show the extrapolated value of m_N/m_ρ. We have used different plotting symbols for the rho fit and plotted the ratio as a function of the nucleon fit. We see that nucleon fits 3, 4 and 10 which have an $m^{1/2}$ term are systematically lower than the other fits. There is also a systematic pattern to the rho fits. From largest to smallest ratio, the order is always fit 12, fit 8 (which are nearly indistinguishable), fit 4, fit 3 and fit 10 (which we could refuse to consider on theoretical grounds). At the conference, two versions of this plot were shown. The first had equal size symbols for each fit, and makes it easy to see the systematics described above. The second version, shown here, has symbol sizes that are proportional to the confidence level of the lattice spacing extrapolation. To have a believable calculation, the chiral and lattice spacing extrapolations must both have good confidence levels. We see that fits 8, 12 and 4 are favored for the rho extrapolation and that the nucleon fits that include the term $m^{1/2}$ are favored. The fits fall close to the experimental value of 1.22, with the (a extrapolation) favored fits falling just slightly below the experimental value. For any particular set of fitting functions, the error in m_N/m_ρ is about 0.02; however, the variation of all the fitting functions is about five times as large. The fits that include a $Q\chi PT$ $m^{1/2}$ term all fall below the experimental value.

Table 4
Lattices used by the IBM group

$6/g^2$	size	#
5.7	$8^3 \times 32$	2439
5.7	$16^3 \times 32$	219
5.7	$24^3 \times 32$	92
5.93	$24^3 \times 36$	210
6.17	$32^2 \times 30 \times 40$	219

3. WILSON SPECTRUM

3.1. Motivation

The most extensive calculation (at least before this conference) of the quenched Wilson spectrum was carried out recently on IBM's GF11 parallel computer by Butler *et al.* [10]. They attempted to control all the sources of systematic error we discussed above. Their ensemble of lattices is displayed in Table 4. They had three lattice sizes at $6/g^2 = 5.7$, but only one at their other two couplings. They also claimed on the basis of their calculations at 5.7 that they could make a finite volume correction to their final answer. They obtained $m_N/m_\rho = 1.278 \pm 0.068$ at finite volume, but with the volume correction, they got 1.216 ± 0.104, in excellent agreement with the observed value 1.222. This celebrated result was based on a combination of three small sink sizes. The sink size parameter r_0 determines the size over which quark fields are smeared. First, the gauge is fixed to Coulomb gauge, then the smeared field ϕ_{r_0} is given by

$$\phi_{r_0}(\vec{x},t) = N \sum_{\vec{y}} \exp(-|\vec{x}-\vec{y}|^2/r_0^2)\psi(\vec{y},t)$$

where N is a normalization factor. (Similarly, a Gaussian distribution can be used to create the source.) They also quote results for a single sink size of 4. The corresponding results 1.328 ± 0.085 and 1.330 ± 0.131 show little finite volume effect and lack the impressive agreement with experiment. In view of the important role of the finite volume correction and the fact that we have a larger ensemble of lattices (except for $N_s = 8$), we decided to do a Wilson spectrum calculation on our $6/g^2 = 5.7$ lattices with $N_s = 12$, 16, 20 and 24. At the time of the conference, we had completed running on the lattices with $N_s = 16$ and 20. We had only 150 $N_s = 12$ and 45 $N_s = 24$ lattices done. All of the running has now been completed, but in the interest of historical accuracy we will only display results that were available during the conference (with one exception).

3.2. Finite size effects

The best available results for the nucleon and rho mass at $6/g^2 = 5.7$ were summarized in Ref. [3], and the graphs also appear as Figs. 10 and 11 at the WWW site mentioned in the introduction. Briefly, the two lightest κ's at which there are results for both $N_s = 16$ and 24 correspond to $m_\pi/m_\rho = 0.69$ and 0.50. Only for the lightest quark mass is there a finite size effect. For the nucleon, it is a 4.8% or 2.6 σ effect. For the rho, it is 3.3% or 2.5 σ. This is certainly a bigger effect than is seen with KS quarks, however; the Wilson masses are all smaller, and if the rho mass is used to set the scale, we would say at $N_s = 16$ (24) the box size is 2.3 (3.4) fm. These differences are based on masses fit with three different sink sizes 0, 1 and 2.

Obviously, the lightest quark mass plays a very important role in the finite size effect seen by the IBM group. In Fig. 9, we compare our results for the nucleon mass at $N_s = 16$ with the IBM group's results at $N_s = 16$ and 24. The mass is shown as a function of the source or sink size parameter. The values from fitting sizes 0, 1 and 2 simultaneously are plotted at size 1/2. We also include a value for $N_s = 20$ that was not available during the conference. These results suggest that the IBM group was really seeing a source size dependence, not a finite size effect. In fact, a source

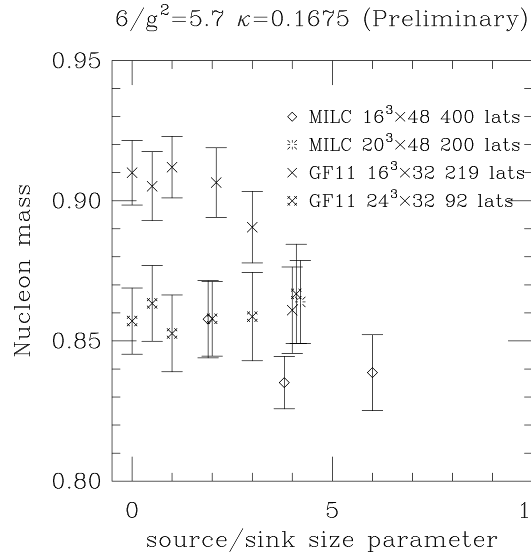

Figure 9. The nucleon mass as a function of source or sink size for $N_s = 16$, 20 and 24 with $6/g^2 = 5.7$.

size dependence is not a physical effect. It can result from not being at a large enough distance from the source to be in the asymptotic regime. Thus, the details of which fits were picked can be important. In Fig. 10, we detail our nucleon mass fits. We fit the nucleon propagator from D_{min} to 10 for $\kappa = 0.1675$. Except for source size 4, we show only single particle fits. For the larger and smaller sources, we attempted some two particle fits, but many did not converge. (As we continue our analysis, we will attempt these again.) Arrows point to the fits that we selected. In each case, we picked the best fit with more than one degree of freedom. The confidence levels are 0.285, 0.421 and 0.427 for source size 2, 4 and 6, respectively. Source size 2 approaches its asymptotic value from above. The fit with $D_{min} = 3$ has confidence level 0.282, so we narrowly missed picking it and having a much larger difference in the masses from the three sources. On the $16^3 \times 32$ lattice, the IBM group used the range 5–8 for sink sizes 0, 1 and combination 0, 1 and 2. For sink size 2, they used 3–7 and for sink sizes 3 and 4, they used 2–5. On the larger

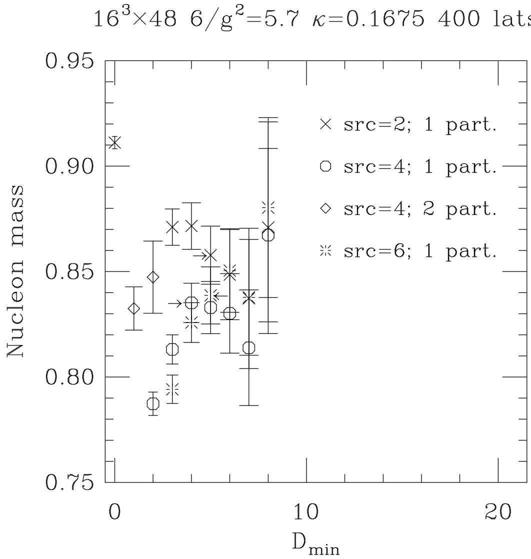

Figure 10. The nucleon mass as a function of minimum distance from source. One and two particle fits are shown. Arrows point to the best fits.

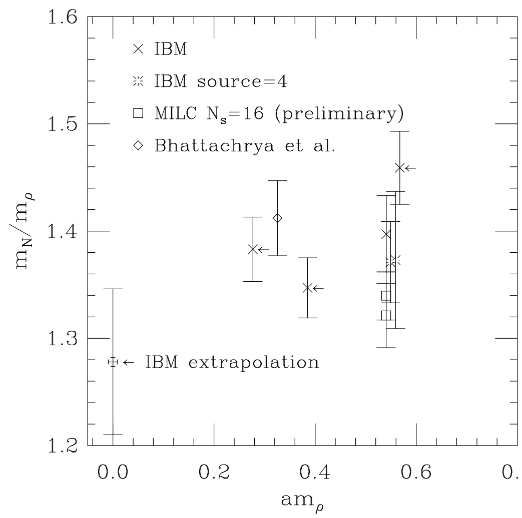

Figure 11. m_N/m_ρ *vs.* am_ρ for Wilson quarks.

lattice, they were a little further from the source, using $D_{min} = 4$–6. Even with these details it is not clear why our results and those of the IBM group differ as much as they do.

3.3. Chiral extrapolations

The reader has just seen that the chiral extrapolation was a major concern for our KS quark calculations. For the IBM group's calculation, chiral extrapolations were based on linear fits to the three lightest quark masses. That corresponds to the range $0.5 \leq m_\pi/m_\rho \leq 0.69$. In our calculations, we have used the same six quark masses on all volumes. For $N_s = 16$ and 24, we have hadrons with non-degenerate quarks. For the mesons, we have 21 combinations of masses.

At the time of the conference, we had just done some simple extrapolations based on the degenerate mass states. Recently, several groups have studied the nonlinear terms that might come into the chiral extrapolations. At Lattice '95, Sloan emphasized that the SCRI calculations with an improved action required nonlinearities for the chiral fits [11]. This was also discussed by Bhattacharya *et al* [12]. But in the latter work, they were not able to distinguish between different higher order terms that appear in χPT, and they did not use the full covariance matrix of the hadron masses in doing the chiral fits, so no reliable confidence level was available. Our fits will be done using the full covariance matrix, just as we did for the KS spectrum. Of particular note is another contribution to this conference by the CP-PACS collaboration, presented by T. Yoshié [13]. In this very high statistics calculation with five quark masses, the nucleon is fit with terms up to cubic in the quark mass and the rho is fit with a quadratic.

3.4. Lattice spacing dependence

With only three values of the gauge coupling, the IBM group's results for the nucleon to rho mass ratio are quite sensitive to each point. If any single point were removed, the result extrapolated to $a = 0$ would be quite different. In Fig. 11, we display the results of the IBM group, and the point from Ref. [12] for $6/g^2 = 6.0$. We also show our preliminary values at 5.7 for $N_s = 16$. There are many points corresponding to the $6/g^2 = 5.7$ and they have been spread out slightly in am_ρ to make it easier to distinguish the points. For the IBM group, we show not only the results for sinks

0, 1 and 2, but their reported result for sink size 4. The two crosses to the right are their multiple sink mass at $N_s = 16$ (upper) and $N_s = 24$ (lower). The two bursts are their sink 4 results for the two volumes. The squares are our preliminary results for source size 4 showing two ways of doing the chiral extrapolation for the nucleon. The diamond is from Ref. [12] where there was also some difficulty in getting the same mass from different sources. The masses used for this point come from a weighted average of three different types of source or sink combinations. The fancy plus is the IBM extrapolated value based on the three crosses with arrows pointing at them. Bhattacharya *et al.*, prefer an extrapolated value close to 1.4.

We are reluctant to redo the a extrapolation at this point. We would like to finish the analysis of our other volumes and do a more careful job of the chiral extrapolation. Further, the talk by Yoshié [13] that followed this one presented evidence that the nucleon mass is "much smaller" than in previous calculations. The couplings used are 5.9, 6.10, 6.25 and 6.47. As these results are likely to supersede prior results, and our small nucleon to rho mass ratio may smoothly join on to their weaker coupling results, it seems unwise to try to convince the reader that the continuum limit quenched Wilson nucleon to rho mass ratio is nearly 1.4, especially when we have evidence that the quenched KS ratio is very near or below the physical value 1.22. (The quenched results for the two types of lattice quarks may not agree with experiment, but we certainly expect them to agree with each other in the continuum limit.) In fact, though CP-PACS did not present a graph with the same axes as Fig. 11, their results for the nucleon mass in the continuum limit are falling below the physical value. From their graph of the spectrum, we estimate that they find $m_N/m_\rho = 1.17 \pm 0.05$. This would be in very reasonable agreement with our KS quark results. (See Fig. 8.)

4. CONCLUSIONS

We have summarized some of our recent results for the quenched spectrum with Kogut-Susskind and Wilson quarks. For the Kogut-Susskind case, we have studied four couplings, with five masses at each coupling. We have found that the chiral extrapolation is the most difficult issue to deal with. Just on the basis of goodness of fit, we have not been able to confirm or refute the presence of artifacts predicted by quenched chiral perturbation theory. If we had been able to determine which chiral extrapolation is correct, our error would have been about 0.02 for the nucleon to rho mass ratio. Without that determination, the systematic error is about five times as large and the ratio is consistent with the experimental value.

We have also done a modest calculation with Wilson quarks at a single coupling, in which we have concentrated on understanding the finite size effects and the systematics of the calculation of Ref. [10]. Our preliminary results show no evidence for finite size effects for $N_s = 16$ with $6/g^2 = 5.7$. Our different source sizes on the $N_s = 16$ lattice give consistent nucleon masses. After the conference, we completed our running on $N_s = 12$, 20 and 24. We have a great deal of analysis to do to extract the hadron masses. Since we have six different quark masses and propagators with non-degenerate quarks, we hope that we will be able to better elucidate the chiral limit than we were able to do for the KS quarks.

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REFERENCES

1. D. Weingarten, Phys. Lett. **109B** (1982) 57; H. Hamber and G. Parisi, Phys. Rev. Lett. **47** (1981) 1792.
2. The MILC Collaboration, talk by D. Toussaint, these proceedings.
3. S. Gottlieb, Nucl. Phys. **B** (Proc. Suppl.) **53** (1997)155.
4. F. Brown, Phys. Rev. Lett. **58** (1987) 2394.
5. N. Cabibbo and E. Marinari, Phys. Lett. **119B** (1982) 387; A. Kennedy and B. Pendleton, Phys. Lett. **145B** (1985) 393.
6. M. Fukugita, *et al.*, Nucl. Phys. **B** (Proc. Suppl.) **20** (1991)376; Phys. Rev. Lett. **68** (1992) 761; Phys. Rev. **D47** (1993) 4739; The MILC Collaboration, C. Bernard *et al.*, Nucl. Phys. **B** (Proc. Suppl.) **26** (1992)262; Nucl. Phys. **B** (Proc. Suppl.) **30** (1993)369; Phys. Rev. **D48** (1993) 4419; Nucl. Phys. **B** (Proc. Suppl.) **34** (1994)366.
7. C. Bernard and M. Golterman, Phys. Rev. **D46** (1992) 853; Nucl. Phys. **B** (Proc. Suppl.) **26** (1992)360; S. R. Sharpe, Phys. Rev. **D41** (1990) 3233; Phys. Rev. **D46** (1992) 3146; J. N. Labrenz and S. R. Sharpe, Nucl. Phys. **B** (Proc. Suppl.) **34** (1994)335.
8. J. Gasser and H. Leutwyler, Phys. Rep. **C87** (1982) 77; Nucl. Phys. **B250** (1985) 465.
9. R. Gupta, Nucl. Phys. **B** (Proc. Suppl.) **42** (1994)85; S. Kim and D. K. Sinclair, Phys. Rev. **D52** (1995) 2614.
10. F. Butler *et al.*, Nucl. Phys. **B430** (1994) 179; Phys. Rev. Lett. **70** (1993) 2849.
11. S. Collins *et al.*, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996)378.
12. T. Bhattacharya, R. Gupta and S. Sharpe, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996)549. T. Bhattacharya *et al.*, Phys. Rev. **D53** (1996) 6486.
13. CP-PACS Collaboration, talk presented by T. Yoshié, these proceedings.